

## Factoring

Step One:

ALWAYS check and see if there is a greatest common factor. If so, then you should factor the greatest common factor out first. You must carry the greatest common factor through the rest of your factoring process. The greatest common factor will include the greatest number which will divide evenly into all the number coefficients of the terms. The greatest common factor will also include any variable and the lowest exponent on the variable if the variable appears in each of the terms of the original polynomial

Examples:

$$3x^2 - 6x = 3x(x - 2)$$

$$x(m - 2) - y(m - 2) = (m - 2)(x - y)$$

Step Two:

Does the polynomial have TWO terms. If so, then try to apply one of the patterns that you must memorize.

\*Difference of Two Squares:

$$x^2 - y^2 = (x - y)(x + y)$$

Example:

$$m^2 - 5^2 = (m - 5)(m + 5)$$

\*Difference of Two Cubes:

$$x^3 + y^3 = (x - y)(x^2 - xy + y^2)$$

Example:

$$B^3 - 2^3 = (B - 2)(B^2 + 2B + 4)$$

\*Sum of Two Cubes

$$X^3 + Y^3 = (X + Y)(X^2 - XY + Y^2)$$

Example:

$$t^3 + 125 = t^3 + 5^3$$

$$= (t + 5)(t^2 - 5t + 25)$$

Step 3:

Does the polynomial have THREE terms? Check and see if there is a 1 on the squared variable. If there is a 1 on the squared variable, then draw skeleton parentheses and try the trial and error method of factoring.

Example:

$$X^2 + 5X + 6 =$$

Draw skeleton parentheses:

$$( \quad ) ( \quad )$$

You need an X positioned in the parentheses to give you an X<sup>2</sup> if the factors were multiplied.

$$(X \quad )(X \quad )$$

Now, you need two numbers which meet these requirements:

\*The two numbers must have a product the same as the number term of the trinomial. (In the example this is the number 6.)

\*The two numbers must have a sum the same as the coefficient of the X (non-squared variable). (In the example, this would be the number 5.) The two numbers meeting these requirements are +2 and +3 and you place these numbers in the skeleton parentheses as shown in example previously.

$$X^2 + 5X + 6 = (X + 2)(X + 3)$$

Note: If the second sign in the original trinomial is positive then the signs of the factors must be the same and will have to be the same as the first sign in the original trinomial. If the second sign in the original trinomial is negative, then the signs in the factors must be different and the sign on the larger number in the factors must be the same as the first sign in the original trinomial.

More Examples:

$$X^2 + 9X + 20 = (X + 4)(X + 5)$$

$$X^2 - 4X - 5 = (X + 1)(X - 5)$$

$$X^2 + 5X - 14 = (X + 7)(X - 2)$$

$$X^2 - 7X + 12 = (X - 3)(X - 4)$$

If the trinomial has a number other than one on the squared variable, then you may also utilize the trial and error method of factoring previously discussed. However, there is also a special method which you may consider using.

Special Method:

(1) Multiply the coefficient of the squared term (usually located in the first position) by the number term (usually located in the third position). Now, you need two numbers which meet the requirements described as follows:

\*The product of the two numbers must be the same as the product you got when you multiplied the coefficient of the squared term times the number term.

\*The sum of the two numbers must be the same as the coefficient of the non-squared variable.

(2) Rewrite the middle term of the original trinomial using the two numbers you have come up with to re-express the middle term as two terms. You now have four terms

(3) Group the first two terms together and the last two terms together. Factor the common factor out of each of the groupings. You now have two terms. Factor the common factor out of the two terms you have just created.

Example:

$$6X^2 + 7X + 2 =$$

You need two numbers with the same product as  $6 \bullet 2$  and with a sum of 7. The two numbers are 4 and 3. Using these two numbers rewrite the polynomial as:

$$6X^2 + 4X + 3X + 2 =$$

Now factor by the grouping procedure used when a polynomial has four terms.

$$6X^2 + 4X + (3X + 2) =$$

$$2X(3X + 2) + (1)(3X + 2) =$$

$$(3X + 2)(2X + 1)$$

Examples:

$$6X^2 - 13X + 6 = (2X - 3)(3X - 2)$$

$$15X^2 + 26X + 8 = (5X + 2)(3X + 4)$$

Step 4:

Does the polynomial have FOUR terms? Try to factor by grouping. Use parentheses to group the first two terms together and the second two terms together. You may have to use the commutative property to move the terms around before you group them. The terms you group together should have a factor in common. When you group the terms together note that you will be isolating the plus or minus sign originally located in the middle of the polynomial. If the sign isolated by the grouping is a negative sign, you must make the necessary changes in the grouping following the negative sign. This will involve changing the sign on each term in the grouping following the negative sign to the opposite of what the sign on the term originally was.

Example:

$$mx+3m+nx+3n$$

Group:

$$(mx + 3m) + (nx + 3n)$$

Factor out the common factor:

$$m(x + 3) + n(x + 3)$$

Again, factor out the common factor:

$$(x + 3)(m + n)$$

Example:

$$ax - ay - bx + by$$

Group, note the sign change in the second grouping which follows after the isolated negative sign:

$$(AX - AY) - (BX - BY) =$$

Factor out the common factor from each grouping:

$$a(x - y) - b(x - y) =$$

Again, factor out the common factor from each term:

$$(x - y)(a - b)$$